

(c) a)

Question 1

$$uX + vY = \begin{cases} (u+v)X & \text{w.p. } \frac{1}{2} \\ (u-v)Y & \text{w.p. } \frac{1}{2} \end{cases}$$

This is because:

$$Y \cdot XZ = \begin{cases} X & \text{if } Z=1 \\ -Y & \text{if } Z=-1 \end{cases} \quad X \sim (0,1) \\ \mu_X(t) = e^{\frac{t^2}{2}}$$

$$E[e^{uX+vY}] = \frac{1}{2} \mu_X(u+v) + \frac{1}{2} (\mu_X(u-v))$$

$$= \frac{e^{\left(\frac{u+v}{2}\right)^2} + e^{\left(\frac{u-v}{2}\right)^2}}{2} = e^{\frac{u^2+v^2}{2}}$$

by expansion

$$= \frac{e^{u^2+v^2+2uv} + e^{u^2+v^2-2uv}}{2}$$

$$= \left( e^{\frac{u^2+v^2}{2}} \right) \left( \frac{e^{uv} + e^{-uv}}{2} \right) \dots (i)$$

b) Choose  $u=0$ , in a above we get

$$E(e^{vY}) = e^{\frac{v^2}{2}} \quad \therefore Y \sim N(0,1)$$

$$Y \sim N(0,1)$$

$$\mu_Y(t) = e^{\frac{t^2}{2}}$$

$$\mu_{X+Y}(u,v) =$$

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$$M_{X,Y}(v,u) = E[e^{ux+uy}] - E(e^{ux} e^{vy})$$

4 MGF of  $\gamma$   $\cdot$   $M_{X,Y}(u,v)$

$$= E(e^{ux}) E(e^{vy}) \cdot \frac{e^{uv} + e^{-uv}}{2}$$

$$= M_X(0) M_X(v) \cdot \frac{e^{uv} + e^{-uv}}{2}$$

then now  $\frac{e^{uv} + e^{-uv}}{2} \neq 1$  since  $u \neq 0$  or  $v \neq 0$

Therefore,  $X$  and  $Y$  are not independent.

4  $X$  and  $Y$  are independent, then

$$M_{X,Y}(v,u) = M_X(u) M_Y(v)$$

We get that

$$M_{X,Y}(u,v) \neq M_X(u) M_Y(v)$$

So  $X, Y$  are not independent.

## Question 2

To show  $\gamma_2$  and  $X$  are not correlated.

$$E(\gamma_2 \cdot X) = E(\gamma_2) \cdot E(X)$$

We start with Left hand side (LHS)

$$E(\gamma_2 \cdot X) = E[\gamma - E(\gamma|X) \cdot X]$$

$$= \cancel{E(X \cdot E(\gamma - E(\gamma|X) \cdot X))}$$

$$= E(E(\gamma - E(\gamma|X) \cdot X | X))$$

$$= E(X \cdot E(\gamma - E(\gamma|X)))$$

$$= E(X \cdot [E(\gamma) - E(E(\gamma|X))])$$

$$= E(X \cdot (E(\gamma) - E(\gamma)))$$

$$E(\gamma_2 \cdot X) = 0$$

Right hand side (RHS)

$$E(\gamma_2) \cdot E(X) = E(\gamma - E(\gamma|X)) \cdot E(X)$$

$$= E(\gamma) - E(\gamma) \cdot EX$$

$$= 0$$

Therefore L.H.S = R.H.S = 0

Hence  $\gamma_2$  and  $X$  are uncorrelated.